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A general theory for temperature dependence in biology

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$$B \propto M^{3/4}$$

Kleiber's law

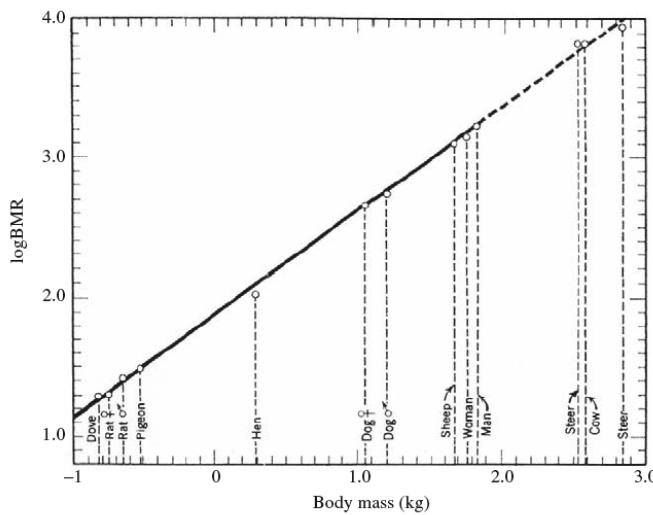
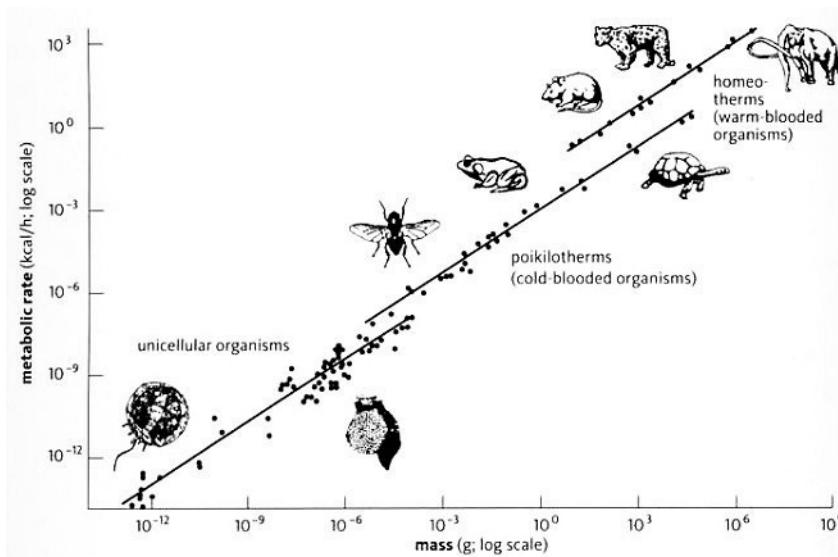


Fig. 1. Kleiber's original 1932 plot of the basal metabolic rate of mammals and birds (in kcal/day) plotted against mass (M_b in kg) on a log-log scale (Kleiber, 1975). The slope of the best straight-line fit is 0.74, illustrating the scaling of metabolic rate as $M_b^{3/4}$. The diameters of the circles represent his estimated errors of 10% in the data.



A General Model for the Origin of Allometric Scaling Laws in Biology

Geoffrey B. West, James H. Brown,* Brian J. Enquist

Allometric scaling relations, including the 3/4 power law for metabolic rates, are characteristic of all organisms and are here derived from a general model that describes how essential materials are transported through space-filling fractal networks of branching tubes. The model assumes that the energy dissipated is minimized and that the terminal tubes do not vary with body size. It provides a complete analysis of scaling relations for mammalian circulatory systems that are in agreement with data. More generally, the model predicts structural and functional properties of vertebrate cardiovascular and respiratory systems, plant vascular systems, insect tracheal tubes, and other distribution networks.

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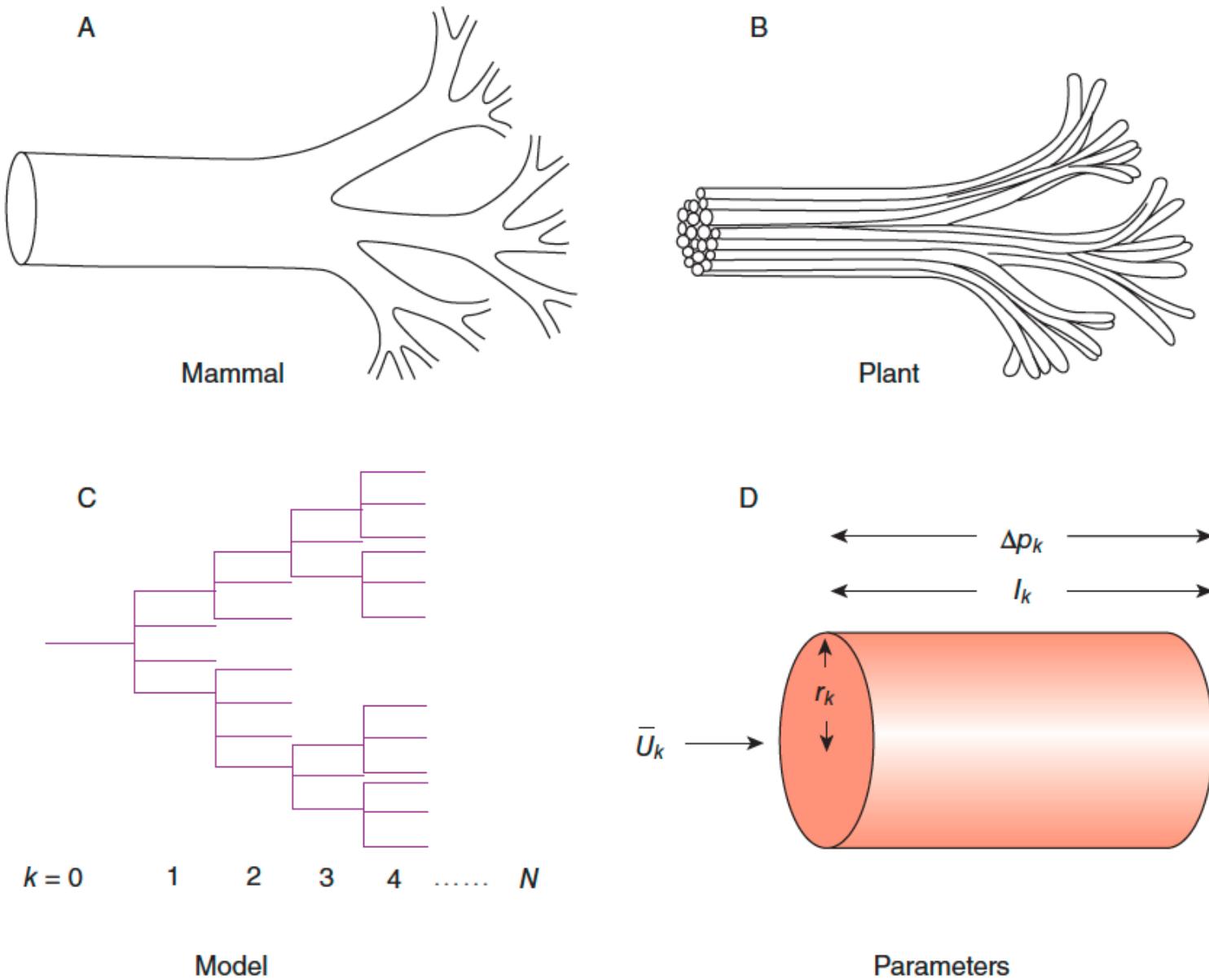


Figure 2.2 Framework for the West et al. (1997) model showing diagrammatic examples of segments of biological distribution networks: (A) mammalian circulatory and respiratory systems composed of branching tubes; (B) plant vessel-bundle vascular system composed of diverging vessel elements; (C) topological representation of such networks, where k specifies the order of the level, beginning with the aorta ($k = 0$) and ending with the capillary ($k = N$); and (D) parameters of a typical tube at the k th level. (From West et al. 1997.)

What is this?

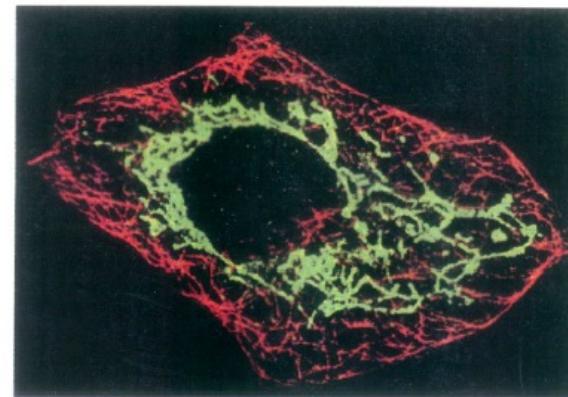
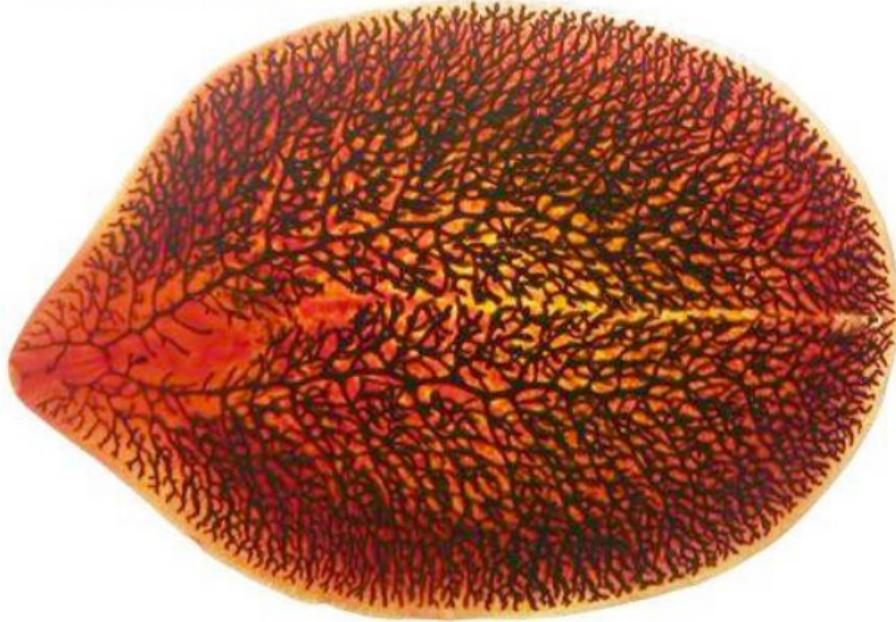
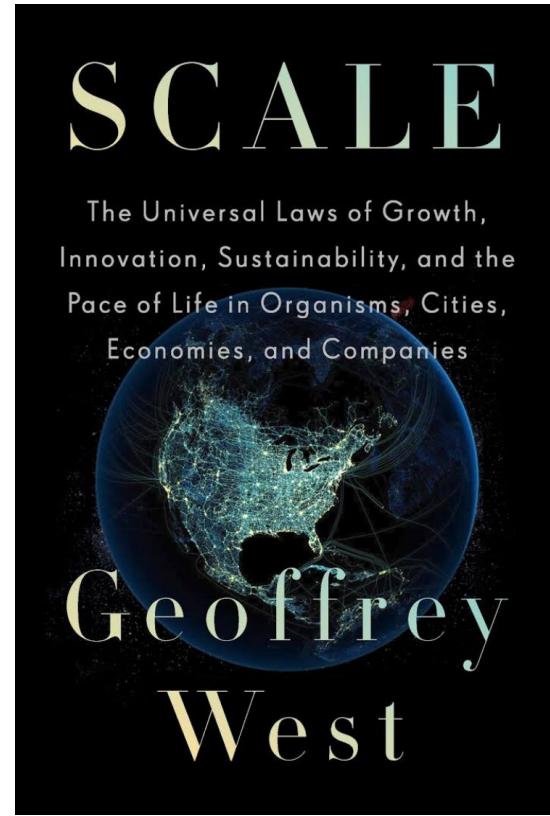
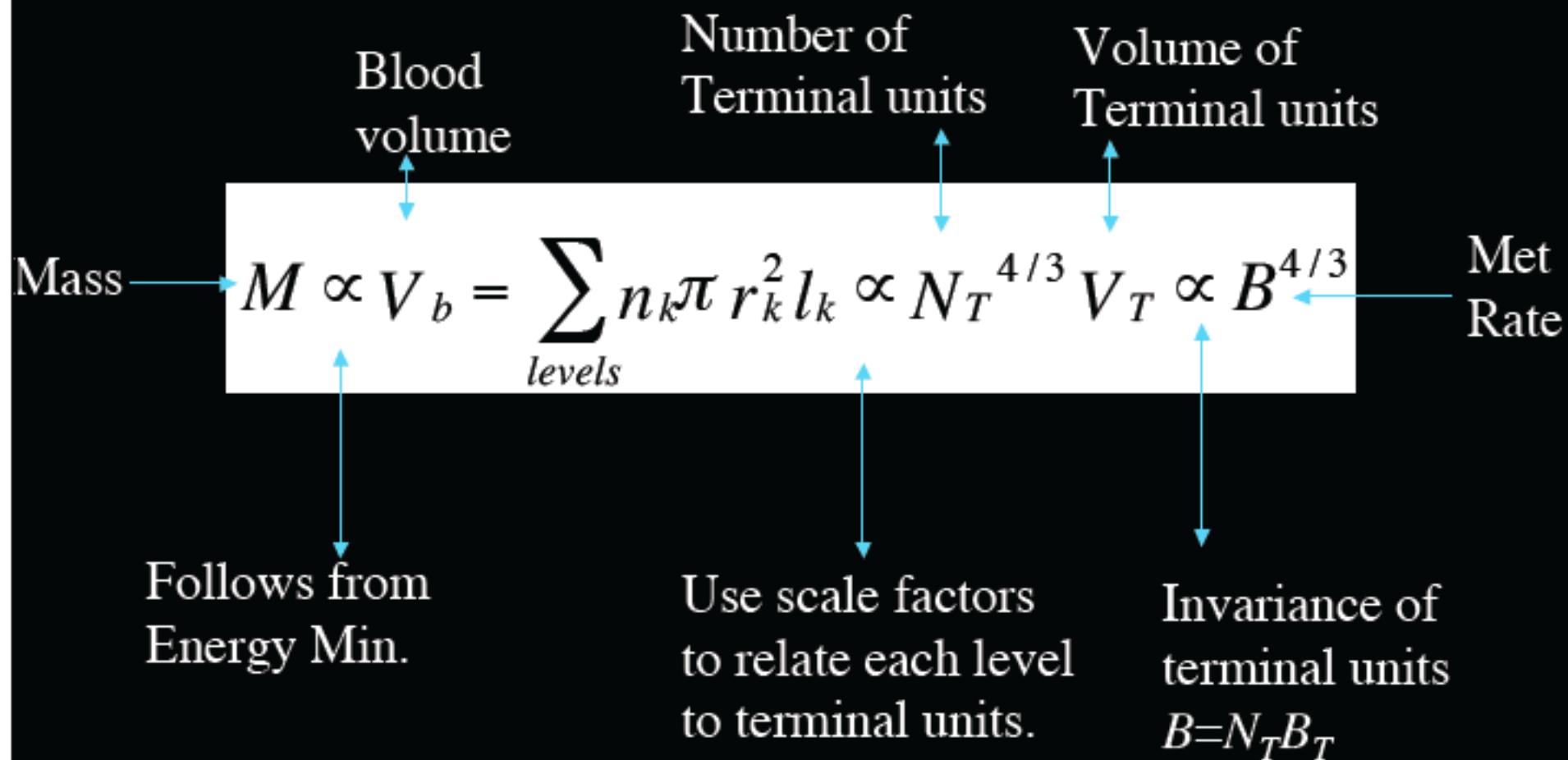


Fig. 1. Mitochondrial network in a mammalian fibroblast. A COS-7 cell labeled to visualize mitochondria (green) and microtubules (red) was analyzed by indirect immunofluorescence confocal microscopy. Mitochondria were labeled with antibodies to the β subunit of the F₁-ATPase and a rhodamine-conjugated secondary antibody. Microtubules were labeled with antibody to tubulin and a fluorescein-conjugated secondary antibody. Pseudocolor was added to the digitized image. Scale: 1 cm = 10 μ m.

From M. P. Yaffe, *Science*, 283, 1493 (1999).



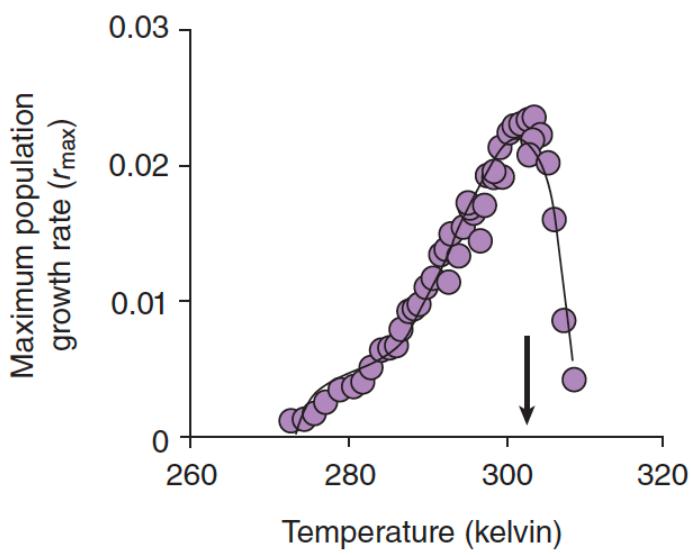
Metabolic Rate, B , and Body Mass, M



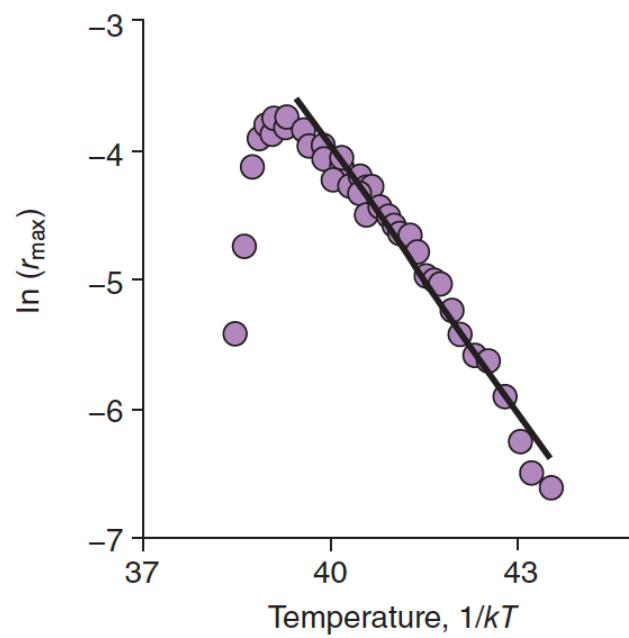
$$B \propto M^{3/4}$$

Temperature and its impact on scaling

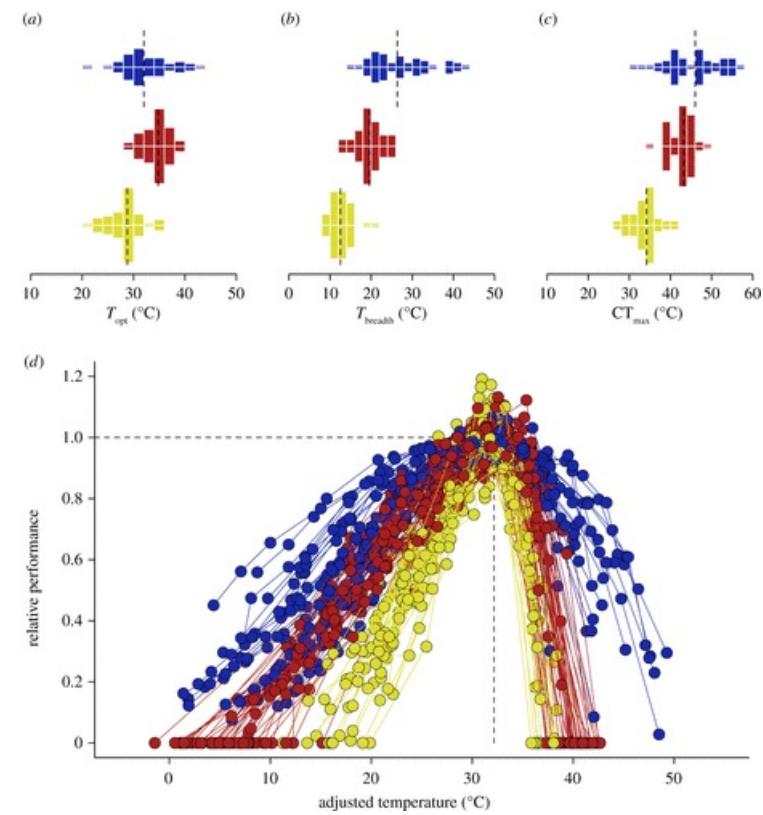
A



B



$$B = B_0 M^\alpha e^{-E/kT}$$



Transition State Theory

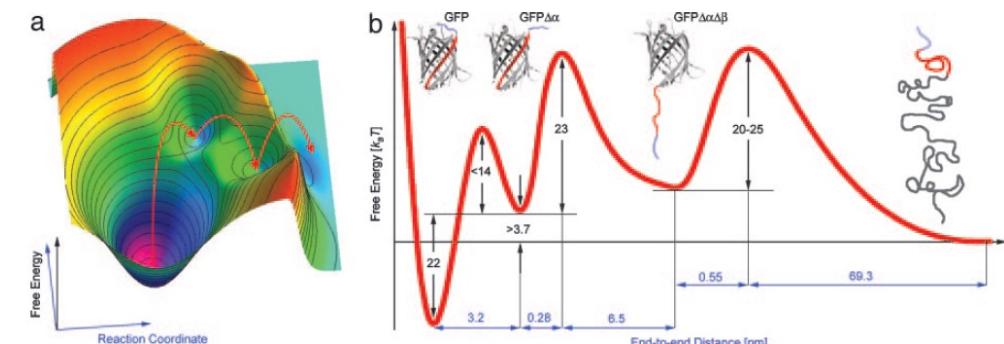
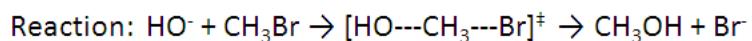
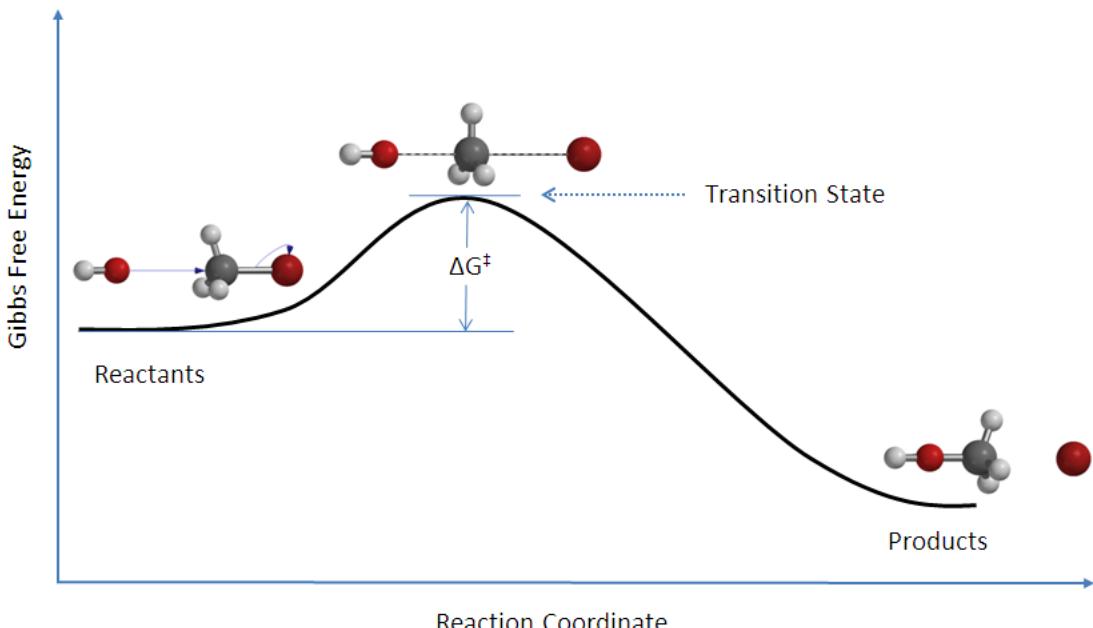
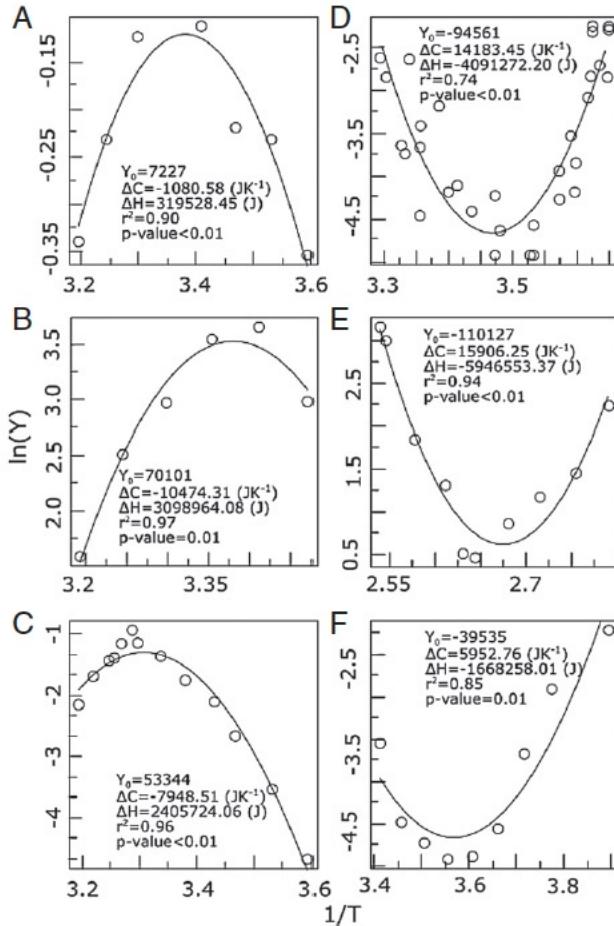


Fig. 6. Free energy surface. (a) Cartoon of the multidimensional energy landscape of GFP. The red arrows indicate the course of the mechanical unfolding pathway. (b) Projection of the energy landscape along the unfolding pathway onto one reaction coordinate.

The Eyring–Evans–Polanyi (EEP) transition state theory (TST)

Metabolism



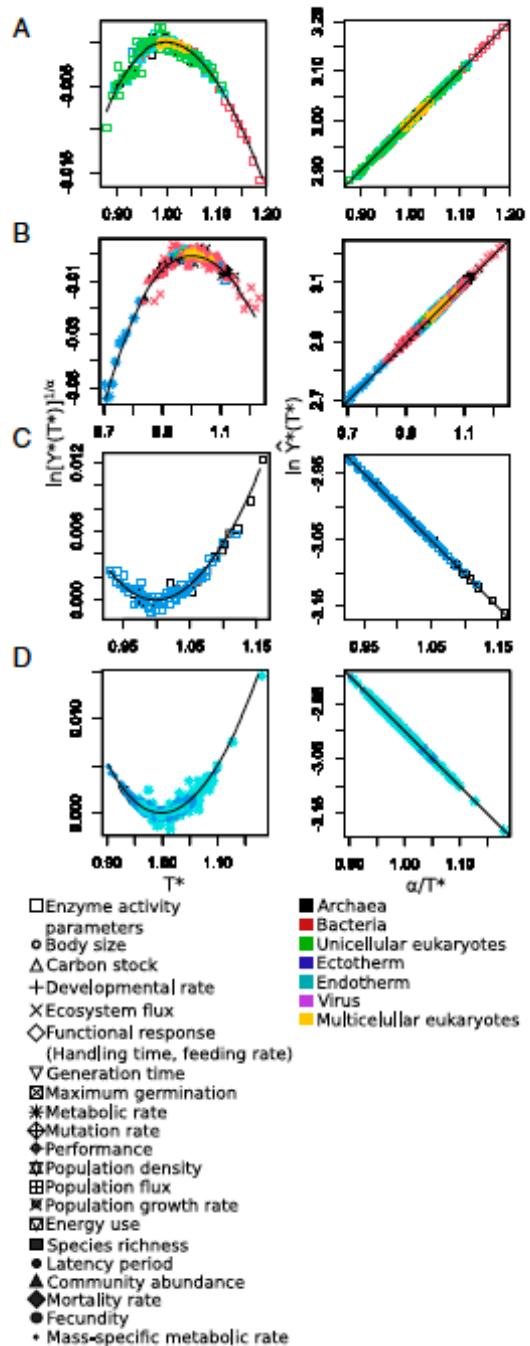
$$k = \frac{k_B}{h} T e^{\Delta S / R} e^{-\Delta H / RT}.$$

Mortality rate

Generation time

Metabolic rate

$$Y(T) \approx Y_0 \left(\frac{1}{T} \right)^{\frac{-\Delta C}{R} - \alpha} e^{\frac{-\Delta H}{RT}}.$$



$$Y^{*1/a} = T^* e^{1/T^*-1},$$

$$\hat{Y}^*(T^*) \equiv (e/T^*)^a Y^*(T^*) = e^{a/T^*}$$